

Non-perturbative dynamics and charge fluctuations in effective chiral models

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Abstract

We discuss the properties of fluctuations of the electric charge in the vicinity of the chiral crossover transition within effective chiral models at finite temperature and vanishing net baryon density. The calculation includes non-perturbative dynamics implemented within the functional renormalization group approach. We study the temperature dependence of the electric charge susceptibilities in the linear sigma model and explore the role of quantum statistics. Within the Polyakov loop extended quark-meson model, we study the influence of the coupling of quarks to mesons and to an effective gluon field on charge fluctuations. We find a clear signal for the chiral crossover transition in the fluctuations of the electric charge. Accordingly, we stress the role of higher order cumulants as probes of criticality related to the restoration of chiral symmetry and deconfinement.

Keywords: QCD phase diagram; Heavy ion collisions; Chiral phase transition; Charge fluctuations; Particle freeze-out; Hadron resonance gas

1. Introduction

Thermodynamics of strongly interacting matter has been explored numerically within Lattice Quantum Chromodynamics (LQCD) [1, 2, 3, 4] as well as experimentally in heavy-ion collisions [5]. The fundamental goal of these studies is to reveal the structure of the QCD phase diagram and to investigate the critical properties of QCD.

LQCD results show that at finite temperatures QCD exhibits restoration of chiral symmetry and deconfinement in a crossover transition. The LQCD equation of state indicates a clear separation between the confined hadronic phase and the deconfined quark–gluon plasma. The temperature separation of the phases of QCD is very transparent when one considers fluctuations of conserved charges [2, 6, 7, 8, 9, 10]. At vanishing chemical potential the variance of both the net baryon number and the electric charge exhibits a rapid change in a narrow temperature interval. This behavior is attributed to the deconfinement of quarks [11, 12, 13]. On the other hand, the fourth order cumulants of

these charges show peak-like structures, whereas the sixth order cumulants are negative in a narrow temperature interval [6].

These properties of the fluctuations observed in LQCD at finite temperature and for small masses of the up- and down-quarks can be attributed to the critical dynamics of the explicitly broken chiral symmetry, which is characterized by the $O(4)$ universality class of QCD [6, 14]. As noted in [14, 15], this could have interesting phenomenological implications: it has been suggested, that the sixth and eight order cumulants of the net baryon number and electric charge may allow an experimental verification of the chiral crossover transition in heavy-ion collisions [15].

The critical behavior of strongly interacting matter related with chiral dynamics should be common to all models which exhibit the underlying chiral symmetry of QCD and the same patterns of spontaneous chiral symmetry breaking at finite temperature. Thus, such effective models have been used to study the thermodynamics near the chiral phase transition and to explore observables which are sensitive to the critical behavior [16, 17, 18, 19, 20, 21, 22, 23]. In this context, models which include the coupling of quarks to an effective gluon field, such as the Polyakov loop extended Nambu–Jona–Lasinio (PNJL) [18, 21] and the quark–meson (PQM) [24] models, are very useful. Both models reproduce the essential features of the QCD thermodynamics already in the mean-field approximation [16, 17, 18, 19, 21]. However, to correctly account for the critical dynamics, it is necessary to go beyond the mean-field approximation and include non-perturbative fluctuations. This can be achieved e.g. by employing methods based on the functional renormalization group (FRG) [25, 26, 27, 28]. The FRG has been applied to study the fluctuations of the net baryon number near the chiral phase transition at finite and at vanishing baryon chemical potential [14, 29, 11, 13, 30]. However, so far such studies are not available for the electric charge fluctuations.

In this work, we study fluctuations of the electric charge near the chiral phase transition at finite temperature and at vanishing chemical potential within the FRG approach to the linear sigma and PQM models. We explore the temperature dependence of these fluctuations and the role of quantum statistics. Furthermore, the influence of quark- and gluon-dynamics on the properties of the electric charge fluctuations near the chiral crossover transition, including the effect of mesonic fluctuations, is discussed. We find a strong sensitivity of higher order cumulants of the electric charge to chiral symmetry breaking and deconfinement. We also show, that these cumulants exhibit a characteristic $O(4)$ structure, similar to that found in the fluctuations of the net baryon charge. In particular, the cumulants turn negative in the vicinity of the chiral transition also at vanishing net charge.

In the subsequent section we introduce the non-perturbative thermodynamic potential in the FRG approach to the PQM model formulated at finite electric-charge chemical potential. In section 3 we present our results on fluctuations of the electric charge, including higher cumulants, in the linear sigma and PQM models. Finally, in section 4 we present our conclusions.

2. The thermodynamic potential in the Polyakov-quark-meson model

The quark-meson model is an effective realization of the low-energy sector of QCD, which incorporates chiral symmetry and exhibits a global $SU(N_c)$ color symmetry. Hence, it does not describe quark confinement. Nevertheless, by introducing a coupling of the quarks to a uniform temporal color gauge field, represented by the Polyakov loop, one obtains an equation of state with properties that closely resemble those of QCD matter, including a change of effective degrees of freedom with increasing temperature, thus mimicking the confinement-deconfinement transition [17, 19, 21, 24]. This is sometimes referred to a *statistical confinement*.

A formulation of the thermodynamics of the PQM model, which remains valid near the chiral phase transition, requires the use of non-perturbative methods. In an analysis of electric charge fluctuations, it is of particular importance to account for mesonic fluctuations, including specifically interacting pions as carriers of electric charge. We employ a method based on the functional renormalization group (FRG) to compute the thermodynamic potential in the PQM model. This method involves an infrared regularization of the fluctuations at a sliding momentum scale k , resulting in a scale-dependent effective action Γ_k [25, 26, 27, 28]. We treat the Polyakov loop as a background field, which is introduced self-consistently on the mean-field level while the quark and meson fields, fluctuations are accounted for by solving the FRG flow equations.

We follow the procedure used in Ref. [29] in the formulation of the flow equation for the scale-dependent grand canonical potential density, $\Omega_k = T\Gamma_k/V$, for the quark and meson subsystems at finite temperature and for a non-vanishing electric charge chemical potential. The thermodynamic potential is obtained by solving the flow equation

$$\begin{aligned} \partial_k \Omega_k(\ell, \ell^*; T, \mu) = & \frac{k^4}{12\pi^2} \left\{ \frac{1}{E_\pi} \left[1 + 2n_B(E_\pi; T) \right] \right. \\ & + \frac{1}{E_\pi} \left[1 + 2n_B(E_\pi - \mu_\pi; T) \right] + \frac{1}{E_\pi} \left[1 + 2n_B(E_\pi + \mu_\pi; T) \right] \\ & \left. + \frac{1}{E_\sigma} \left[1 + 2n_B(E_\sigma; T) \right] - \sum_{f=u,d} \frac{4N_c}{E_q} \left[1 - N(\ell, \ell^*; T, \mu_f) - \bar{N}(\ell, \ell^*; T, \mu_f) \right] \right\}. \end{aligned} \quad (1)$$

Here $n_B(E; T)$ is the bosonic distribution function

$$n_B(E; T) = \frac{1}{\exp(E/T) - 1},$$

$\mu_\pi = e_\pi \mu_Q$ is the charge pion chemical potential and $e_\pi = 1$ the charge of a π^+ . The pion and sigma energies are given by

$$E_\pi = \sqrt{k^2 + \overline{\Omega}'_k}, \quad E_\sigma = \sqrt{k^2 + \overline{\Omega}'_k + 2\rho \overline{\Omega}''_k},$$

where the primes denote derivatives with respect to $\rho = (\sigma^2 + \vec{\pi}^2)/2$ of $\bar{\Omega} = \Omega + c\sigma$. The fermion distribution functions $N(\ell, \ell^*; T, \mu_f)$ and $\bar{N}(\ell, \ell^*; T, \mu_f)$,

$$N(\ell, \ell^*; T, \mu_f) = \frac{1 + 2\ell^* e^{\beta(E_q - \mu_f)} + \ell e^{2\beta(E_q - \mu_f)}}{1 + 3\ell e^{2\beta(E_q - \mu_f)} + 3\ell^* e^{\beta(E_q - \mu_f)} + e^{3\beta(E_q - \mu_f)}}, \quad (2)$$

$$\bar{N}(\ell, \ell^*; T, \mu_f) = N(\ell^*, \ell; T, -\mu_f), \quad (3)$$

are modified because of the coupling to the gluon field. Finally, the quark energy reads

$$E_q = \sqrt{k^2 + 2g^2\rho} \quad (4)$$

and the quark chemical potentials are defined by

$$\mu_u = \frac{1}{3}\mu_B + e_u\mu_Q, \quad \mu_d = \frac{1}{3}\mu_B + e_d\mu_Q \quad (5)$$

with $e_u = 2/3$ and $e_d = -1/3$.

The flow equation (1) is solved numerically with the ultraviolet cutoff $\Lambda = 1.2$ GeV using the polynomial method described in Ref. [29]. In this scheme, the stationarity condition

$$\left. \frac{d\Omega_k}{d\sigma} \right|_{\sigma=\sigma_k} = \left. \frac{d\bar{\Omega}_k}{d\sigma} \right|_{\sigma=\sigma_k} - c = 0 \quad (6)$$

is implemented in the flow equation. The initial conditions for the flow are chosen to reproduce the following vacuum properties: the physical pion mass $m_\pi = 138$ MeV, the pion decay constant $f_\pi = 93$ MeV, the sigma mass $m_\sigma = 600$ MeV, and the constituent quark mass $m_q = 300$ MeV at the scale $k \rightarrow 0$. The symmetry breaking term, $c = m_\pi^2 f_\pi$, is treated as an external field which does not flow. The flow of the Yukawa coupling g is neglected because it is not expected to be significant for the present studies (see e.g. Refs. [31]).

By solving Eq. (1) one obtains the thermodynamic potential $\Omega_{k \rightarrow 0}(\ell, \ell^*; T, \mu)$ as a function of the Polyakov loop variables ℓ and ℓ^* . The full thermodynamic potential $\Omega(\ell, \ell^*; T, \mu)$ in the PQM model, including quark, meson and gluon degrees of freedom, is obtained by adding the effective gluon potential $\mathcal{U}(\ell, \ell^*)$,

$$\Omega(\ell, \ell^*; T, \mu) = \Omega_{k \rightarrow 0}(\ell, \ell^*; T, \mu) + \mathcal{U}(\ell, \ell^*). \quad (7)$$

At a given temperature and chemical potential, the Polyakov loop variables, ℓ and ℓ^* , are then determined by the stationarity conditions:

$$\frac{\partial}{\partial \ell} \Omega(\ell, \ell^*; T, \mu) = 0, \quad \frac{\partial}{\partial \ell^*} \Omega(\ell, \ell^*; T, \mu) = 0. \quad (8)$$

The thermodynamic potential (7) does not contain contributions of thermal modes with momenta larger than the cutoff Λ . In order to obtain the correct high-temperature behavior of the thermodynamic functions, we supplement the FRG potential with the contribution of the high-momentum states. A procedure for implementing this was proposed in Ref. [32] for the QM model and extended to the PQM model in Ref. [29], by including the flow of quarks interacting with the Polyakov loop for momenta $k > \Lambda$.

3. Electric charge density fluctuations

The thermodynamic potential obtained by solving the flow equation (1) correctly reproduces the critical O(4) scaling near the chiral phase transition. Thus, using this potential one can compute the temperature dependence of the electric charge fluctuations and their higher cumulants.

The fluctuations of the electric charge are characterized by the generalized susceptibilities,

$$\chi_n^Q(T) = \frac{\partial^n [p(T, \mu_Q)/T^4]}{\partial (\mu_Q/T)^n}. \quad (9)$$

The first cumulant, $\chi_1^Q = n_Q/T^3$, is given by the electric charge density $n_Q = N_Q/V$, while the second cumulant

$$\chi_2^Q = \frac{1}{VT^3} \langle (\delta N_Q)^2 \rangle \quad (10)$$

with $\delta N_Q = N_Q - \langle N_Q \rangle$, is proportional to the variance of the electric charge. Furthermore, the fourth and sixth order cumulants can be expressed through moments of δN_Q

$$\chi_4^Q = \frac{1}{VT^3} [\langle (\delta N_Q)^4 \rangle - 3\langle (\delta N_Q)^2 \rangle^2], \quad (11)$$

$$\begin{aligned} \chi_6^Q &= \frac{1}{VT^3} [\langle (\delta N_Q)^6 \rangle - 15\langle (\delta N_Q)^4 \rangle \langle (\delta N_Q)^2 \rangle - 10\langle (\delta N_Q)^3 \rangle^2 \\ &+ 30\langle (\delta N_Q)^2 \rangle^3]. \end{aligned} \quad (12)$$

Based on LQCD results [33], one expects that in the region of the crossover transition, the fluctuations of the electric charge should reflect the critical scaling of the underlying O(4) symmetry. For the O(4) universality class, the singular part of thermodynamic pressure, at small values of the baryon chemical potential, scales as

$$p \propto (T/T_c - 1 + \kappa \mu_B^2)^{2-\alpha}, \quad (13)$$

where T_c is the temperature of the second-order phase transition at $\mu = 0$ in the chiral limit. The exponent $\alpha \approx -0.21$ is the critical exponent of the specific heat in the O(4) universality class in three-dimensions. Therefore, at vanishing chemical potential, the cumulants χ_n^B of the net baryon number diverge at the transition temperature for all even $n \geq 6$,

$$\chi_{2n}^B \propto (T/T_c - 1)^{2-n-\alpha}. \quad (14)$$

Fluctuations of the electric charge are related to the net baryon and isospin fluctuations by

$$\chi_n^Q = \frac{1}{2^n} \left[\chi_n^B + \chi_n^I + \sum_{i=1}^{n-1} \binom{i}{n} \frac{\partial^n (p\beta^4)}{\partial (\beta\mu_I)^i \partial (\beta\mu_B)^{n-i}} \right]. \quad (15)$$

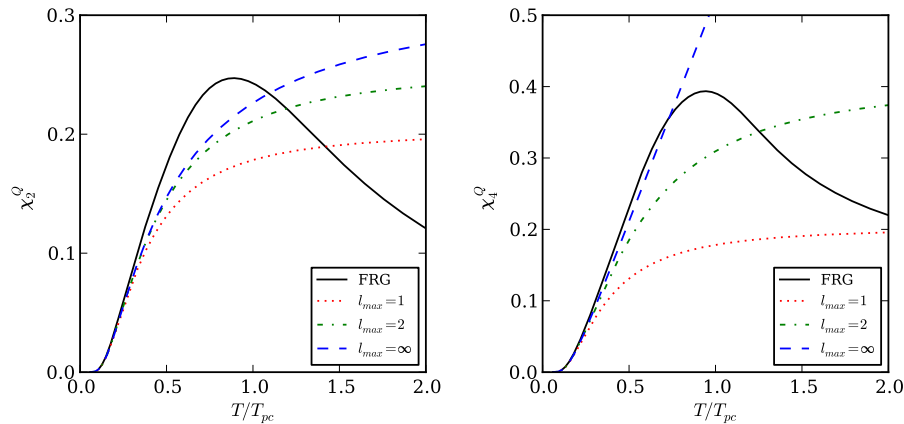


Figure 1: (Color online) The second χ_2^Q and the fourth χ_4^Q order cumulant of the net electric charge fluctuations in the linear sigma model obtained within the functional renormalisation group approach (solid lines). The dotted line is the contribution of an ideal pion gas in the Boltzmann approximation. The dash-dotted ($l_{max} = 1$) and dashed ($l_{max} = 2$) lines show the first and the second terms in the series (17), respectively. The temperature axis is normalized by $T_{pc} = 180.5$ MeV which is the pseudo-critical temperature in the model within the FRG approach.

The isovector fluctuations are regular at the chiral phase transition in an isospin symmetric system. Thus, the χ_n^I remain finite at the phase transition. The last term, which correlates isospin with baryon number, can diverge for sufficiently large n . However, these terms give subleading contributions to criticality of χ_n^Q . Thus, the leading singular part of the electric-charge cumulants χ_n^Q is solely determined by χ_n^B . Consequently, at small quark masses, the contributions from the singular part of the thermodynamic pressure to the net electric charge fluctuations should be similar to that found for the net baryon fluctuations.

The expected residual $O(4)$ scaling in χ_n^B at finite quark masses is indeed observed in the PQM model in the FRG approach. However, since the regular contributions to the electric charge and to the baryon number fluctuations are different, it is not *a priori* clear that this will be the case also for cumulants χ_n^Q of the electric charge.

In the following, we compute the temperature dependence of χ_n^Q in the PQM model with the thermodynamic potential obtained from Eq. (7). We also discuss the contributions of individual modes to the properties of χ_n^Q near the chiral transition.

3.1. Fluctuations in an interacting pion gas

We first consider the contribution of mesonic degrees of freedom to the fluctuations of the net electric charge. Clearly, in such studies pionic interactions should be included. A suitable model for exploring the meson contribution is the mesonic linear sigma model. The thermodynamic potential for this model

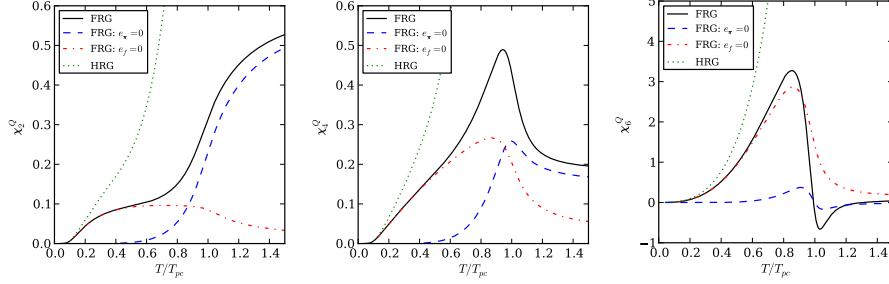


Figure 2: (Color online) The cumulants of the electric charge fluctuations in the Polyakov loop extended quark-meson model calculated within the functional renormalisation group approach (solid lines). The results are also shown for only charged quarks $e_\pi = 0$ and only charged pions $e_f = 0$ (see text). The hadron resonance gas contribution is shown as dotted lines.

is obtained directly from the flow equation (7) in the limit of $N_c \rightarrow 0$. The fluctuations of the electric charge are then due only to the charged pion degrees of freedom.

In the mean-field approximation to the linear sigma model, fluctuations of the electric charge are absent. They can be included perturbatively [34, 35, 36] or non-perturbatively within the FRG approach, where fluctuations due to interacting pions are included [11, 12, 29, 37]. Here we employ the FRG approach to account for meson fluctuations.

The second and the fourth order cumulants obtained in the FRG approach are shown in Fig. 1. Although the chiral symmetry is explicitly broken by the vacuum pion mass, a clear signature of the chiral crossover transition is seen in the susceptibilities. In particular, we note the non-monotonic temperature dependence of the fourth order cumulant, resulting in a peaked structure near the chiral transition. This behavior is due mostly to the temperature dependence of the pion mass, which up to the chiral transition remains close to its vacuum value, while above the transition m_π grows strongly with temperature. Consequently, above the pseudo-critical temperature (T_{pc}), all cumulants of the net-charge fluctuations decrease with temperature. Note that this effect is not a consequence of the cut off, which suppresses high momentum modes in the model. Due to the large value of $\Lambda = 1.2$ GeV, this suppression sets in only at much higher temperatures.

It is interesting to assess at which temperature pionic interactions start influencing the charge fluctuations. This can be estimated by comparing the FRG results for the χ_2^Q and χ_4^Q with those obtained for an ideal gas of charged pions. The pressure of the latter is given by

$$\frac{p_{\pi^+} + p_{\pi^-}}{T^4} = \frac{m^2}{2\pi^2 T^2} \sum_{l=1}^{\infty} \frac{1}{l^2} K_2(l\beta m) (e^{l\beta\mu_q} + e^{-l\beta\mu_q}), \quad (16)$$

where $K_2(x)$ is a modified Bessel function and μ_q is the electric chemical potential. In an ideal pion gas, the odd-order cumulants of the electric charge vanish

at $\mu_q = 0$, while even cumulants are given by

$$\chi_n^Q = \frac{m^2}{\pi^2 T^2} \sum_{l=1}^{\infty} l^{n-2} K_2(l\beta m). \quad (17)$$

This can be considered as an approximation to the regular part of the fluctuations of the electric charge.

In Fig. 1 the contributions of the regular part to the second and fourth cumulants are shown. At low temperatures the FRG and ideal pion gas results agree. This indicates that the FRG method correctly accounts for the pionic contribution to the thermodynamics.

The effect of quantum statistics can be studied by truncating the series in Eq. (17) at a given order $l = l_{max}$. This is illustrated in Fig. 1, where we confront the $l_{max} = 1$ and 2 truncations for the second and the fourth order cumulants of the free pion gas with the full ($l_{max} = \infty$) result. Clearly, the leading contribution, which corresponds to Boltzmann statistics, deviates considerably from the full quantum statistics results, except at very low temperatures. The higher-order terms in the series contribute effectively as multiply charged particles. Hence, their contribution increases with the order of the cumulant. This expectation is confirmed by Fig. 1, which shows that the deviation is stronger for χ_4^Q than for χ_2^Q . Thus, the effect of quantum statistics cannot be neglected in the calculation of fluctuations of the electric charge. This is a reflection of the fact that multi-charged particles can yield a dominant contribution to the fluctuations of conserved charges [6]. In QCD this is the case for fluctuations of the electric charge and strangeness, where double- and triple-charged baryons play an important role.

3.2. Electric charge cumulants in the PQM model

The dynamics of the PQM model and its particle content is more relevant for QCD than the mesonic linear sigma model described above. The PQM model contains charged-quarks as dynamical degrees of freedom and exhibits statistical confinement through the coupling of the quarks to the effective gluon field.

In Fig. 2 we show the first three non-trivial cumulants χ_2^Q , χ_4^Q and χ_6^Q of the electric charge for vanishing baryon- and electric charge chemical potential obtained from Eq. (7). We also show separately the contributions of quarks or pions. This separation is made by setting the electric charge of pions e_π and then that of quarks e_f to zero, respectively.

The second-order cumulant χ_2^Q shows a rapid increase near the pseudo-critical temperature, interpolating between the pion and quark contributions. Since χ_2^Q is not influenced by the singular part of the thermodynamic pressure (13), the behavior seen in Fig. 2 is due to the “statistical confinement” of the PQM model. The rapid increase with temperature is a consequence of the rapid unleashing of single- and double-quarks states at the crossover temperature. The pion contribution, on the other hand, is suppressed above T_{pc} , owing to the increasing thermal mass.

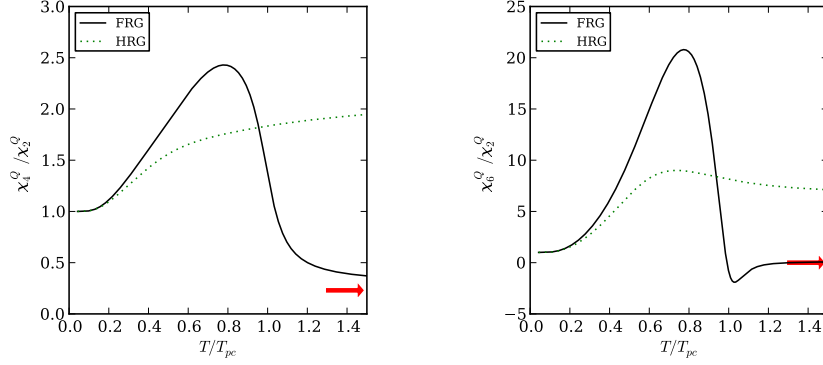


Figure 3: (Color online) The kurtosis (χ_4^Q/χ_2^Q) and the (χ_6^Q/χ_2^Q) ratio calculated in the Polyakov loop extended quark-meson model. The arrow shows the corresponding Stefan-Boltzmann limits. The hadron resonance gas model results are indicated as dotted-lines.

The fourth- and sixth-order cumulants exhibit a peak, which increases in strength with the order of the cumulant. This can be understood in terms of the critical dynamics of the chiral transition; with increasing order of the cumulant, the contribution of the singular part of the pressure (13) becomes more and more dominant. The negative structure of the χ_6^Q near T_{pc} is similar to that observed for the sixth order cumulant of net baryon number fluctuations and is due to the particular form of the $O(4)$ scaling function [14]. The peak in χ_4^Q appears as a sum of quark and pion contributions, while the peak in χ_6^Q is dominantly due to pions. The negative region of χ_6^Q near T_{pc} is due to the quark contribution.

In Fig. 2 we also compare the PQM results with that of the hadron resonance gas (HRG) model, which includes the contributions of all charged hadrons and resonances. The HRG model reproduces the thermodynamics of LQCD in the hadronic phase up to $T \simeq 0.9T_{pc}$.

There is good agreement between the PQM and HRG model results at low temperatures, where in both cases pions are the dominant degrees of freedom. For higher temperatures the HRG overshoots the PQM model results. Thus, the PQM model does not yield a quantitative description of the LQCD results on thermodynamics. Nevertheless, this model can provide useful insights into the critical dynamics of universal quantities near the chiral phase transition.

In order to reduce the contribution of the non-singular part to the fluctuations and to focus on the critical behavior, ratios of χ_n^Q to the second-order cumulant χ_2^Q , which is not influenced by the critical chiral dynamics at $\mu = 0$ [6], have been studied. In Fig. 3 we show the temperature dependence of the kurtosis $\kappa = \chi_4^Q/\chi_2^Q$ and of the ratio χ_6^Q/χ_2^Q near the chiral crossover transition. At low temperatures, $\kappa \rightarrow 1$, as expected for a non-interacting Boltzmann gas. For higher temperatures, the kurtosis increases owing to the contribution

of double-charged baryons and to higher order quantum corrections to the pion distribution. Near T_{pc} and above the quarks dominate, because of the increasing pion mass, resulting in κ decreasing towards the Stefan-Boltzmann limit of free quark gas. The ratio χ_6^Q/χ_2^Q exhibits a much stronger peak, which diverges at the critical temperature in the chiral limit. In analogy with the net baryon fluctuations [14, 15], this ratio also develops a negative region near T_{pc} , owing to the chiral critical dynamics encoded in the $O(4)$ scaling function.

As shown in Fig. 2, in the PQM model both ratios χ_4^Q/χ_2^Q and χ_6^Q/χ_2^Q overshoot the HRG model predictions. This indicates that in the HRG model, the contribution of multi-charged states to fluctuations is reduced relative to that of singly-charged particles more than in the PQM model.

4. Summary and conclusions

We have computed the electric charge fluctuations in the vicinity of the chiral crossover transition within effective chiral models. Our calculations include non-perturbative dynamics implemented within the functional renormalization group approach at finite temperature and at vanishing baryon density. We have performed our studies in the linear sigma model and in the Polyakov-loop extended quark-meson model.

We find a strong sensitivity of the fluctuations of the electric charge to the chiral transition. We have also explored the influence of multi-charged states and quantum statistics on the temperature dependence of cumulants of the electric charge.

Our results show that at physical pion and quark masses, the higher order cumulants of the electric charge fluctuations exhibit properties which can be linked to the chiral critical dynamics expected in the $O(4)$ universality class. The qualitative structure of fluctuations in the PQM model is similar to that obtained in lattice QCD. This confirms the phenomenological importance of electric charge fluctuations, in particular the sixth- and higher-order cumulants, as signatures for the chiral crossover transition in heavy-ion collisions.

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